Generating New Hyperbolic Solutions for Nonlinear Physical Model by Tanh Method

Scientific Modelling and Research
Vol. 7, No. 1, 1-6, 2022
e-ISSN: 2523-952X

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ABSTRACT

Tanh method is used in this paper to find hyperbolic solutions to ion sound and Langmuir wave systems describes ion sound wave and the Langmuir wave as part of the action of the ponder motive force caused by a high-frequency field. A power series in tanh method was used as an ansatz to obtain analytical solutions for certain nonlinear evolution equations of traveling wave type. The suggested method, is a powerful solution method and it based on wave transformation, converts the partial derivatives in the equation to ordinary derivatives and then generate new hyperbolic solutions to the system under consideration in this paper. Graphical simulations are introduced for some solutions. The method's main aspects will be discussed and the results showed the strength this using method through the new solutions obtained using tanh method. This method generates entirely new solutions for other types of nonlinear evolution equations observed in physics.

Keywords: *Algebraic equations, Graphical solution, Hyperbolic solutions, Ion sound and langmuir waves systems, Nonlinear partial differential equations, Tanh method.*

DOI: 10.55284/smr.v7i1.704
Citation | Ammar Al-Salih (2022). Generating New Hyperbolic Solutions for Nonlinear Physical Model by Tanh Method. Scientific Modelling and Research, 7(1): 1-6.

Funding: This study received no specific financial support.
Competing Interests: The author declares that there are no conflicts of interests regarding the publication of this paper.
History: Received: 20 July 2022/ Revised: 24 August 2022 / Accepted: 8 September 2022/ Published: 30 September 2022
Publisher: Online Science Publishing

Highlights of this paper

- We obtain new hyperbolic solutions to ion sound and Langmuir wave systems.
- Tanh method is efficient and we can applied for different physical problems to reach exact solutions.

1. INTRODUCTION

The importance of nonlinear partial differential equations cannot be affected and applied to a wide range of phenomena and dynamic processes in physics, chemistry, biology, fluid dynamics, plasma, optical fibers, and other engineering fields. So, there was great interest and effort from researchers to study nonlinear partial differential equations. Searching for exact solutions to these equations by suggesting new methods that represent a solution to physical and engineering problems. The availability of symbolic computation software, such as Maple, which is able to solve nonlinear partial differential equations. So, exact solution which obtained from different methods to the nonlinear equations have become very important resulting in methods like exp-function method [1, 2] sine- cosine method [3, 4] Jacobi elliptic function method [5, 6] extended tanh function method [7-9] and there in. The exact solutions obtained using these methods, as well as the types of solutions obtained, such as solitary wave solutions, shock wave solutions, periodic wave solutions, and others. We used tanh method [10, 11] in this article to generate hyperbolic exact solutions to NPDEs. employing a traveling wave transformation, these nonlinear partial differential equations transform it to a group of algebraic equations and then solve it gives hyperbolic solutions of NPDEs.. By using a traveling wave transformation, these nonlinear partial differential equations transform it to a group of algebraic equations and then solve it gives hyperbolic solutions of NPDEs. To understand this method, we investigate [12] system to ion sound and Langmuir waves. This system describes ion sound wave and the Langmuir wave as part of the action of the ponder motive force caused by a high-frequency field. Previously, various researchers used different methods to obtain new hyperbolic solutions to the system ion sound and Langmuir wave, such as Baskonus and Bulut [13]; Hassan and Abdelrahman [14]. This study organized as: Tanh method described in Section 2. We applied tanh method to the system for the ion sound and Langmuir waves and obtained exact solutions, including hyperbolic wave solution, to demonstrate the method In Section 3. Finally, in Section 4, we present our conclusions..

2. DESCRIPTION THE TANH METHOD

Tanh method proposed in Wazwaz [15]; Wazwaz [16]; Wazwaz [17]; Wazwaz [18]; Hosseini, et al. [3]; Malfliet [10]. The tanh method dependent on traveling wave solutions and we can write this method was developed by Malfliet [10] to solve the coupled KdV equations by tanh function.

Consider system of nonlinear partial differential equations:

$$H_1(u, v, u_t, v_t, u_x, v_x, u_{xx}, v_{xx}, \dots) = 0$$
$$H_2(u, v, u_t, v_t, u_x, v_x, u_{xx}, v_{xx}, \dots) = 0 \quad (1)$$

Where H_1, H_2 are variables' polynomials u, v as well as their derivatives. Let $(x,t) = U(\xi)$ and $v(x,t) = V(\xi)$ where $\xi = k(x - \lambda t)$. Here k and λ are constant.By using chain rule we get:

$$\frac{\partial}{\partial t} = -k\lambda \frac{d}{d\xi}, \quad \frac{\partial}{\partial x} = k\frac{d}{d\xi}, \quad \frac{\partial^2}{\partial x^2} = k^2\frac{d^2}{d\xi^2}, \quad \frac{\partial^3}{\partial x^3} = k^3\frac{d^3}{d\xi^3}, \dots \quad (2)$$

Then Equation 1 become ordinary differential equations:

$$P_1(U, U', U'', U''', \dots, V, V', V'', V''', \dots) = 0$$
$$P_2(U, U', U'', U''', \dots, V, V', V'', V''', \dots) = 0 \quad (3)$$

With P_1, P_2 are polynomials which represent reduced ordinary differential Equations 3. Integrate Equation 3 and make constant integration equal to zero in solutions nonzero constants, on the other hand, can be used and handled [15]. The exact solution for Equation 1 is now found to be equivalent to the obtained solutions from reduced ordinary differential Equation 3. We introduce a new independent variable for the tanh method:

$$u\,(x,t) \;=\; tanh(\xi) \tag{4}$$

Which results in a variable change in the derivatives:

$$\frac{d}{d\xi} = (1 - Y^2)\frac{d}{dY},$$

$$\frac{d^2}{d\xi^2} = -2Y(1 - Y^2)\frac{d}{dY} + (1 - Y^2)^2\frac{d^2}{dY^2},$$

$$\frac{d^3}{d\xi^3} = \; 2(1 - Y^2)(3Y^2 - 1)\frac{d}{dY} - 6Y(1 - Y^2)^2\frac{d^2}{dY^2} + (1 - Y^2)^3\frac{d^3}{dY^3} \tag{5}$$

In similar way, we can derive the other derivatives. The next step is that the solutionwe are wanted to find it expressed in the form:

$$u(x,t) = U(\xi) = \sum_{i=0}^{m} a_i\, Y^i$$
$$v(x,t) = V(\xi) = \sum_{i=0}^{n} b_i\, Y^i \tag{6}$$

Now, we balance highest order linear term with the nonlinear terms to find the parameter m and n in Equation 1, and k, λ, a_0, a_1, … , a_m, b_0, b_1, … , b_n are to be determined. Substituting (6) in Equation 3 will provide an algebraic equation set for k, λ, a_0, a_1, … , a_m, b_0, b_1, … , b_n because all Y^i coefficients must vanish for $i = 0,1,2,3$ …. then find values of these coefficients by using Maple software, and using(6), we obtain solutions for $u(x,t)$ and $v(x,t)$ in new form [18].

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3. APPLICATIONS

We are now employing the tanh method to solve the system of ion sound and Langmuir waves.

The ion sound and Langmuir wave systems are written as follows [12]:

$$iE_t + \frac{1}{2}E_{xx} - VE = 0,$$

$$V_{tt} - V_{xx} - 2(|E|^2) = 0 \tag{7}$$

Where Ee^{iwp^t} is the normalized Langmuir oscillation's electric field [14].

We use travelling wave transformation for Equation 7 as follows:

$$(x,t) = e^{iz}E\,(\xi), \quad (x,t) = (\xi) \;\; z = \gamma x + \lambda t\;, \xi = \alpha x + \beta t \tag{8}$$

Where γ, λ, α and β are constants. Then, put Equation 8 with derivatives in Equation 7 we get:

$$i\,(\beta + \alpha\gamma)' = 0, \tag{9}$$
$$\alpha^2 E'' - (2\lambda + \gamma^2)\,E - 2EV = 0, \tag{10}$$
$$(\beta^2 - \alpha^2)V'' - 2\alpha^2(E^2)'' = 0 \tag{11}$$

Equation 11 integrating twice to ξ and put zero to constant of integration, we have:

$$V(\xi) = \frac{2\alpha^2}{\beta^2 - \alpha^2}E^2 \tag{12}$$

Put Equation 12 in Equation 10 and also Equation 9, get:

$$\alpha^2(\gamma^2 - 1)E'' - (\gamma^2 - 1)(2\lambda + \gamma^2)E - 4E^3 = 0 \tag{13}$$

Now, we introduce $Y = tanh(\xi)$ and by using (5), Equation 13 becomes:

$$\alpha^2(Y^2 - 1)(1 - Y^2)^2\frac{d^2E}{dY^2} - 2\alpha^2(\gamma^2 - 1)Y(1 - Y^2)\frac{dE}{dY} - (\gamma^2 - 1)(2\lambda + \gamma^2)E - 4E^3 = 0 \tag{14}$$

Now, to find m, We find that balancing the highest order linear term with the highest order nonlinear term yields $m = 1$,

The tanh method become as:

$$E(x,t) = E(Y) = a_0 + a_1 Y \ \ , a_1 \neq 0 \tag{15}$$

Substituting E, E', E'' from Equation 15 into Equations 14, then multiplying by the coefficient of Y^i, $i = 0, 1, 2, 3$ obtain a nonlinear algebraic equation system:

Y^0: $4a^3 + (\gamma^2 - 1)(2\lambda + \gamma^2)a_0 = 0$

Y^1: $2\alpha^2(\gamma^2 - 1)a_0 + (\gamma^2 - 1)(2\lambda + \gamma^2)a_1 + 12a^2\ a_1 = 0$ 0

0

Y^2: $12a^2 a_0 = 0$

1

$$Y^3: 4a^3 - 2\alpha^2(\gamma^2 - 1)a_0 = 0 \tag{16}$$

1

By using maple solve the nonlinear systems of Equation 16 we get:

$$a_0 = 0\ , \qquad \gamma = 1 \tag{17}$$

From (17) we get new solution to Equation 7:

$E\,(x,t) = a_1 e^{i(x+\lambda t)}\tanh(\alpha x + \beta t)$,

$$V\,(x,t) = \frac{2\alpha^2 a_1^2}{\beta^2 - \alpha^2}\tanh^2(\alpha x + \beta t) \tag{18}$$

The hyperbolic solutions for $E\,(x,t)$ and (x,t) are shown in Figure 1 for some fixed parameter values, ($\lambda = 1$, $\alpha = 1$, $\beta = 1$, $a_1 = 1$)

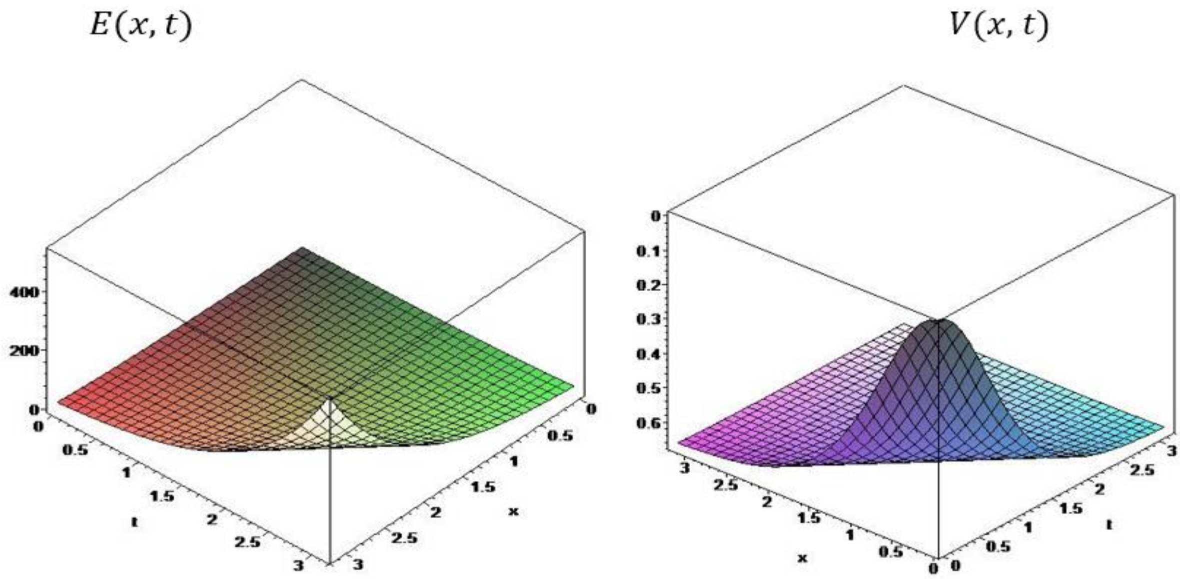

Figure 1. Hyperbolic waves solutions for Equation 18.

4. CONCLUSIONS

We successfully applied the tanh method to the system of ion sound and Langmuir waves and generate new solutions for this system, proving that the tanh method can be used to obtain exact solutions of nonlinear partial differential equations and that it is a good and efficient method. This method can be applied to find more exact solutions to various types of equations.

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